Phd Thesis (with Master Internship)
Tensor networks for the estimation of high-dimensional distributions on manifolds

Project description

The approximation of high-dimensional functions is a typical task in statistics and machine learning. This requires the introduction of suitable model classes that exploit specific structures of the functions. Model classes of rank-structured functions are among the most prominent tools in data analysis, signal processing and numerical analysis. Among these tools, tree-based tensor formats, or tree tensor networks, have recently attracted a lot of attention [Nou17,CLO16,CPZ17]. These model classes are particular cases of deep neural networks [KNO18] with a particular architecture which provides nice properties from theoretical and practical viewpoints.

In this thesis, we will consider the problem of the approximation of a probability distribution supported on a submanifold of a high-dimensional space, from samples of the distribution. A natural path would be to separate the problem into two tasks: the estimation of the manifold (support of the distribution) and the estimation of the distribution on the manifold. A first objective is to revisit these tasks by using tree tensor networks. A second objective is to address both tasks simultaneously by considering compositions of tree tensor networks.

We will address theoretical questions about the approximation properties of tree tensor networks, based on regularity assumptions on the manifold and the probability distribution. Also, we will address the questions of well-posedness and stability of learning problems. Finally, we will propose and analyze learning algorithms (inspired from [GNC19]) with robust model selection methods based on estimates of the complexity of the considered model classes [Mas07,BMM12].

These contributions will involve tools from different fields such as approximation, geometry, statistics and optimization.

We are seeking for candidates having a strong background in mathematics and a specialization in one or more of the above specific fields.

References

Nou17 A. Nouy. Low-rank methods for high-dimensional approximation and model order reduction. In P. Benner, A. Cohen, M. Ohlberger, and K. Willcox, editors, Model Reduction and Approximation: Theory and Algorithms. SIAM, Philadelphia, PA, 2017.

CLO16 A. Cichocki, N. Lee, I. Oseledets, A. H. Phan, Q. Zhao, and D. Mandic. Tensor networks for dimensionality reduction and large-scale optimization: Part 1 low-rank tensor decompositions. Foundations and Trends in Machine Learning, 9(4-5):249–429, 2016.

CPZ17 A. Cichocki, A.H. Phan, Q. Zhao, N. Lee, I. Oseledets, M. Sugiyama, and D. Mandic. Tensor networks for dimensionality reduction and large-scale optimization: Part 2 applications and future perspectives. Foundations and Trends in Machine Learning, 9(6):431–673, 2017.

KNO18 V. Khrulkov, A. Novikov, and I. Oseledets. Expressive power of recurrent neural networks. In International Conference on Learning Representations, 2018.

GNC19 E. Grelier, A. Nouy, and M. Chevreuil. Learning with tree-based tensor formats. ArXiv e-prints, 2019.

Mas07 P. Massart. Concentration inequalities and model selection. 2007.

BMM12 J.P. Baudry, C. Maugis and B. Michel. (2012). Slope heuristics: overview and implementation. Statistics and Computing, 22(2), 455-470.

Details

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